MATH 127
MIDTERM II
April 4, 2007

NAME (please print legibly): ____________________________________________
Your University ID Number: ____________________________________________

• No calculators or notes are allowed on this exam.

• Please show all your work. You may use backs of pages if necessary. You
  may not receive full credit for a correct answer if there is no work shown.

• You do not need to simplify all the way.

Unacceptable answer: $4x^2 - x|_1^2$.
Acceptable answer: $(4(2^2) - 2) - (4 - 1)$.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>VALUE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. (25 pts) Let $M$ denote the mass of the earth. The earth exerts a gravitational force of

$$F(r) = \frac{GMm}{r^2} \text{ Newtons}$$

on an object of mass $m$ which is $r$ meters away from the center of the earth. (Treat $G, M, m$ as constants in this problem.)

Let $r_e$ denote the radius of the earth (a constant).

Find the work required to move the object from the surface of the earth ($r = r_e$) to a point infinitely far from Earth ($r = \infty$). (Ignore the gravitational pull of other astronomical bodies.) A correct solution must involve taking a limit!
2. (25 pts) Find the volume of the solid obtained by rotating the shaded region around the $x$-axis. (Begin by deciding whether to use $dx$ or $dy$.)
3. (25 pts) A rock is ejected from the ground upward at an angle of $\pi/3$ radians, at a velocity of 10 ft/sec. Neglect air resistance and use $-32$ as the acceleration due to gravity.

(a) Find an equation for the height at time $t$.

(b) How far away does the rock land?
4. (25 pts) A block of Blackbearium has initial mass 1 kg. Let \( m = m(t) \) represent the mass of the block at time \( t \) (in years). The rate of change of the mass at time \( t \) years is \( \frac{4t}{3m^3} \) kg/yr.

(a) Find the formula for \( m(t) \).

(b) When (in years) will the mass equal 2 kg?