(1) Let $F$ be a field of characteristic $p > 0$, and suppose $E$ is a finite extension. Show that if $p 
mid [E : F]$, then $E/F$ is a separable extension.

(2) Prove that $F_5[\sqrt{2}] \cong F_5[\sqrt{3}]$. (Find an explicit isomorphism.)

(3) Prove that $\mathbb{Q}[\sqrt{2}] \not\cong \mathbb{Q}[\sqrt{3}]$.

(4) Compute $\text{Gal}(\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q}(\sqrt{2}))$ and $\text{Gal}(\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q})$.

(5) Let $E$ be the splitting field inside $\mathbb{C}$ of the polynomial

$$x^4 - 4x^2 + 2 \in \mathbb{Q}[x].$$

Show that $\text{Gal}(E/\mathbb{Q})$ is cyclic of order 4.