(1) Let $G$ be a group with $|G| = 4$. Prove that $G$ is abelian.

(2) Let $A, B$ be subgroups of a group $G$. Prove or give a counterexample:
   (a) $A \cap B$ is a subgroup of $G$.
   (b) $A \cup B$ is a subgroup of $G$.

(3) Let $G$ be a finite abelian group and $a, b \in G$. Let $n = |a|$ and $m = |b|$, and suppose $\gcd(n, m) = 1$. Prove that $|ab| = nm$. Does the conclusion still hold if $G$ is nonabelian? Prove or give a counterexample.

(4) Let $G$ be a group and let $a, b \in G$. Prove that $|ab| = |ba|$.

(5) Let $p$ be a prime. Compute $|\text{GL}_2(\mathbb{Z}/p\mathbb{Z})|$. (Hint: How many possible top rows are there? Given a top row, how many allowable bottom rows are there?) Generalize to find $|\text{GL}_n(\mathbb{Z}/p\mathbb{Z})|$.

(6) Let $S$ be a nonempty subset of a group $G$. The subgroup $\langle S \rangle$ generated by $S$ is $\langle S \rangle = \{x_1x_2\cdots x_n | x_i \in S \text{ or } x_i^{-1} \in S \}$. Prove that $\langle S \rangle = \bigcap_{H \leq G, \ S \subseteq H} H$. 