(1) Suppose $\rho$ is a representation of $S_3$ whose character has the values

\[ \chi_\rho(e) = 7, \quad \chi_\rho((12)) = -1, \quad \chi_\rho((123)) = 4. \]

Find the decomposition of $\rho$ into a direct sum of irreducibles.

(2) Compute the character table of $A_4$.

Hints: Let

\[ H = \{ e, (1\ 2)(3\ 4), (1\ 4)(2\ 3), (1\ 3)(2\ 4) \}. \]

Show that $H$ is a normal subgroup. (This is a one-liner if you observe that $H$ is the union of two conjugacy classes of $S_4$!) Conclude that $A_4/H \cong \mathbb{Z}/3\mathbb{Z}$, and use this to construct three distinct 1-dimensional representations of $A_4$. Next, show that $A_4$ acts 2-transitively on $\{1, 2, 3, 4\}$ to get another irreducible representation.