

MAT 261 HOMEWORK 7: DUE FRIDAY, OCT. 27

- (1) Let  $X = \{1, 2, 3, 4\}$ .
- (a) Write down the equality relation on  $X$  and give the partition of  $X$  induced by it.
- (b) In the table below are some relations on  $X$ . Indicate which are reflexive, symmetric, transitive, antisymmetric, and/or equivalence relations. For each "No" answer, give a reason.

$R$	Ref?	Symm?	Trans?	Antisymm?	Equiv. Rel.?
$\{(1, 1), (1, 2), (2, 2)\}$					
$\{(1, 2), (2, 3), (3, 4), (4, 1)\}$					
$\{(1, 1), (2, 2), (3, 3), (1, 4), (4, 1)\}$					
$\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1)\}$					
$\{(1, 2), (3, 1), (2, 3), (2, 1)\}$					

- (2) Below are three relations on the set  $\mathbf{R}$  of real numbers. Determine whether each is an equivalence relation. If so, carefully prove it, and describe the equivalence classes. If not, give an explanation.
- (a)  $aRb$  if  $a \geq b$ .
- (b)  $aRb$  if  $ab > 0$ . Also do the same problem if the underlying set is  $\mathbf{R}^*$  (the set of nonzero real numbers).
- (c)  $aRb$  if  $|a - b| < 4$ .
- (3) Let  $A$  be a nonempty set, and let  $R$  be a relation on  $A$  which is both symmetric and antisymmetric. Must  $R$  be the equality relation? Analyze the following "proof":

Let  $x, y \in A$ . Since  $R$  is symmetric,  $xRy \implies yRx$ . Since  $R$  is antisymmetric and  $xRy$  and  $yRx$ , we may conclude that  $x = y$ . Therefore  $xRy$  if and only if  $x = y$ , so  $R$  is the equality relation.