

MAT 261 HOMEWORK 5: DUE FRIDAY, OCT. 6

- (1) For any integer  $n$ , let  $n\mathbf{Z} = \{na \mid a \in \mathbf{Z}\}$  be the set of all integer multiples of  $n$ . Prove that

$$6\mathbf{Z} = 2\mathbf{Z} \cap 3\mathbf{Z}.$$

- (2) Prove that for any positive real numbers  $x, y$ ,

$$\frac{x}{y} + \frac{y}{x} \geq 2.$$

(Please do not start your proof by assuming the conclusion! Follow the model from class where we proved  $\frac{x+y}{2} \geq \sqrt{xy}$ .)

- (3) Prove that for any real number  $x$ , there exists a unique real number  $y$  satisfying

$$x^2y = x - y.$$

- (4) Prove that for any  $n \geq 1$ ,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

- (5) (Not collected) Read section 6.4 about using proof by contradiction with caution. Often other methods are shorter, and hence clearer. Continue doing lots and lots of other problems on your own (do odd exercises from the book).