

MAT 261 HOMEWORK 4: DUE FRIDAY, SEPT. 29

- (1) (Note: you are not being asked to prove anything in this problem.)
Consider the following statement:

If x is an even integer, then $x^3 + 3x^2 - 6$ is an even integer.

- (a) State the hypothesis you would adopt if attempting a direct proof of the statement. What conclusion would you seek?
- (b) State the hypothesis you would adopt if attempting a contrapositive proof of the statement. What conclusion would you seek?
- (c) Which method is preferable in this case? Why?
- (2) Prove the following theorem:
Theorem. *If $x \in \mathbf{Z}$, then $x^2 - 2x + 9$ is odd if and only if x is even.*
(When deciding what to do, bear in mind the lesson of the previous problem.)

- (3) Prove that $\sqrt{3}$ is irrational.

- (4) Determine whether or not $\sqrt{2} + \sqrt{6}$ is irrational. Justify your answer.

- (5) Read section 5.3. Read it again.