MAT 261 HOMEWORK 2: DUE FRIDAY, SEPT. 15

In class, we proved the first of DeMorgan's two laws, namely

$$(A \cup B)^c = A^c \cap B^c,$$

arguing as follows. Suppose $x \in (A \cup B)^c$. Then $x \notin A \cup B$, i.e., x belongs to neither A nor B. This means that $x \in A^c$ and $x \in B^c$, i.e., $x \in A^c \cap B^c$. This proves that

$$(A \cup B)^c \subseteq A^c \cap B^c. \tag{1}$$

Conversely, if $x \in A^c \cap B^c$, then $x \notin A$ and $x \notin B$, which says exactly that $x \notin A \cup B$. Hence $x \in (A \cup B)^c$, which proves that

$$A^c \cap B^c \subseteq (A \cup B)^c. \tag{2}$$

From (1) and (2), we conclude that $(A \cup B)^c = A^c \cap B^c$.

It feels like we went in a circle doing the two containments. That's because every step is reversible. It is permissible in such cases to do both directions at once, as follows. We wish to prove that

$$(A \cup B)^c = A^c \cap B^c.$$

Note that

$$\begin{aligned} x \in (A \cup B)^c &\iff x \notin A \cup B \\ &\iff x \notin A \text{ and } x \notin B \\ &\iff x \in A^c \text{ and } x \in B^c \\ &\iff x \in A^c \cap B^c. \end{aligned}$$

From the above, we conclude that the two sets $(A \cup B)^c$ and $A^c \cap B^c$ have the same elements, and hence they are equal.

Assigned problems:

(1) Mimic the above argument to give a 4-line bidirectional proof of the other DeMorgan's Law for sets, namely

$$(A \cap B)^c = A^c \cup B^c.$$

(2) Find the power set of $A = \{1, \{2, \{3\}\}\}$.

(3) Let A, B and C be sets, with $C \neq \emptyset$. Suppose

$$A \times C = B \times C.$$

Prove that $A = B$. (Be sure you clearly convey where the hypothesis $C \neq \emptyset$ is used!)

(4) In the previous problem, does the same conclusion hold if $C = \emptyset$?

(5) Do the following on your own, but don't hand in (check your answers in the back of the book):

p. 7: 1-11 odds, 17-21 odds, 29, 31.
p. 10: 1-11 odds
p. 14: 1-9 odds
p. 16: 1-5 odds, 13, 15
p. 18: 1abc
p. 20: 1ab
p. 23: 11, 13.