MAT 261 HOMEWORK 12: Due **Monday**, Dec. 4

HW12 rewrites will be due on Friday, Dec. 8 (last day of class).

RECALL: For sets A and B, recall that $|A| \leq |B|$ means that there is an injection from A to B. The Schröder-Bernstein Theorem says that if $|A| \leq |B|$ and $|B| \leq |A|$, then $A \sim B$, i.e., there is a bijection from A to B.

ASSIGNED PROBLEMS:

(1) Use induction to prove that for any finite collection of countable sets $A_1, \ldots, A_n$, the Cartesian product $A_1 \times \cdots \times A_n$ is countable.

(2) Show that $\mathbf{N}$ can be partitioned as the disjoint union

$$\mathbf{N} = \cup_{n=1}^{\infty} A_n$$

of an infinite number of infinite subsets A_n.

(3) Use the Schröder-Bernstein theorem to prove that the open unit interval of the real line is equivalent to the half-open unit interval, i.e., $(0,1) \sim (0,1]$.
Comment: It is surprisingly difficult to actually produce a bijection (see the next problem), but the S-B theorem tells you quite easily that one exists.

(4) Let A be an infinite set, and suppose $x \notin A$. Prove that

$$A \sim (A \cup \{x\}).$$

(So, for example, this gives a different proof of the previous problem: $(0,1) \sim [0,1) \sim [0,1]$.)
Hints: First prove it in the case where A is denumerable. For the general case, problem (b) on HW11 will be helpful.