MAT 261 HOMEWORK 11: DUE **Monday**, NOV. 27

Rewrites will be due on Friday, Dec. 1.

We state here for reference the following theorems from class:

Theorem A. *For a nonempty set A, the following statements are equivalent.*

(1) *A is countable.*
(2) *There is a surjection $f : \mathbf{N} \longrightarrow A$.*
(3) *There is an injection $g : A \longrightarrow \mathbf{N}$.*

Theorem C. *The union of a countable collection of countable sets is countable.*

ASSIGNED PROBLEMS:

(a) Let $S = \{n^2 \mid n \in \mathbf{N}\}$ be the set of perfect squares of integers. Prove that S is denumerable.

(b) Prove that every infinite set A has a denumerable subset.
(This is a little bit harder than it sounds. Prove by induction that for every $n \in \mathbf{N}$, A has a subset B_n of size n. Then explain why the subset $B = \bigcup_n B_n$ is denumerable.)

(c) Prove the following generalization of Theorem A:

Theorem B. *For a nonempty set A, the following statements are equivalent.*

(1) *A is countable.*
(2′) *There is a countable set B and a surjection $f : B \longrightarrow A$.*
(3′) *There is a countable set C and an injection $g : A \longrightarrow C$.*

(With the notation of Theorem A, it suffices to prove that (2) $\iff$ (2′) and (3) $\iff$ (3′).)