MAT 261 HOMEWORK 10: DUE **Friday**, NOV. 17

Recall the useful theorem proven in class: If A and B are finite sets of the same size, then a map $f : A \to B$ is injective if and only it if is surjective.

(1) Give counter-examples to show that neither direction of the above bidirectional statement holds true if A and B are allowed to be infinite sets. Feel free to use Theorem 13.1, which says $\mathbf{Z}$ and $\mathbf{N}$ have the same size. (We'll prove this in class on 11/15.)

(2) Define a function $f : \mathbf{Z}/7\mathbf{Z} \longrightarrow \mathbf{Z}/7\mathbf{Z}$ by $f([x]) = [3x - 2]$. Prove in two ways that f is a bijection. First do it by giving a table showing the values of f. Second, do it by proving from the definitions that it is either 1-1 or onto, and then citing the above "useful theorem".

(3) Read section 12.6 and solve problem 6 on page 216.