

MAT 261 HOMEWORK 1: DUE FRIDAY, SEPT. 8

You are most welcome to collaborate with each other on HW, and ask me for help. However, your write-ups must be done on your own. If, on a problem, you are asked a question, you will get no credit for simply answering the question. You need to explain why your answer is correct. Use complete sentences, but at the same time be concise- every line you write should advance the argument.

- (1) Prove that there are infinitely many prime numbers.
- (2) How many positive numbers  $p$  have the property that  $p$  and  $p + 1$  are both prime?
- (3) The *Twin Prime Conjecture* asserts that there are infinitely many positive numbers  $p$  with the property that  $p$  and  $p + 2$  are both prime. (This is an open problem.) But the following is one you can answer: How many positive numbers  $p$  have the property that  $p$ ,  $p + 2$ , and  $p + 4$  are all prime? (Hint: the answer and method of proof are quite similar to those of the previous problem.)
- (4) Find all positive integers  $n$  for which  $3^n - 1$  is prime.