MAT 136 MIDTERM 2 REVIEW

Midterm 2 is Thurs, November 16. Please arrange a 2-hour block of time.

Review of topics covered:

Midterm 2 covers Chapters 7-11, including the following topics:

Continuity: Know what it means for f to be continuous at a point x, in both of the following formulations:

- The function f is continuous at a if $\lim_{x \to a} f(x) = f(a)$.
- The function f is continuous at a if for all $\varepsilon > 0$ there exists $\delta > 0$ such that
  $|f(x) - f(a)| < \varepsilon$ whenever $|x - a| < \delta$.

(Note that we do not say $0 < |x - a| < \delta$ as in the definition of limit, because we need to allow $x = a$.) This is Chapter 6 material, but it wasn't covered on Midterm 1.

Chapters 7-8: Know, and be able to use, the definitions of upper bound, least upper bound, supremum, lower bound, greatest lower bound, and infimum of a subset of $\mathbf{R}$.
Know and apply the completeness axiom of the real numbers (P13).
Be able to prove Theorem 7-1.
Know the statements and consequences of Theorems 7-2 and 7-3.

Chapter 9: Definition of derivative of a function f at a given point a. Interpertation of $f'(a)$ as the instantaneous rate of change of f, the slope of the tangent line at $(a, f(a))$ (geometric), and the velocity of an object if $f(t)$ represents the object's position on the number line at time t (physical).

Chapter 10: Rules of differentiation:

$$(f+g)' = f' + g',$$

$$(cf)' = c(f') \qquad \text{(if } c \text{ is a constant)},$$

power rule, product rule, quotient rule, chain rule, trig derivatives, derivative of e^x.

Chapter 11: Significance of the derivative. Local maximum, local minimum (know the definitions of these terms), critical point (unlike Spivak, we say c is a critical point of f if either $f'(c) = 0$ or $f'(c)$ DNE).

State and prove Fermat's Theorem: If c is a local max or local min, then c is a critical point.

Know the statements of Rolle's Theorem and the Mean Value Theorem. Use the MVT to prove that if $f'(x) = 0$ everywhere in an interval, then f is constant on the interval (Corollary 10-1).

Second derivative, and meaning of concave up, concave down, inflection point.

First Derivative Test and Second Derivative Test (determining the nature of a critical point as a max, a min, or neither).

Sketch the graph of a function f by (1) finding and plotting critical points, (2) finding intervals where f is increasing / decreasing, using the First Derivative Test, (3) finding intervals where f is concave up / concave down, (4) end behavior, i.e. behavior of $f(x)$ as $x \to \infty$ and $x \to -\infty$, (5) behavior near any points where f is undefined.

Chapter 12: We will not cover inverse functions on the midterm, but implicit differentiation is fair game (we did it in class last Wednesday).

1. Sample Problems

(1) Prove that the function $f(x) = x^2$ is continuous at the point 2.

(2) (Problem 6=15) Suppose f is a function and $\lim_{x\to a^+} f(x) = f(a) < 0$. Prove that there exists $\delta > 0$ such that $f(x) < 0$ for all x satisfying $a \leq x < a + \delta$. (This is a one-sided version of Theorem 6-3.)

(3) Prove Theorem 7-1.

(4) (Spivak, Chapter 8, problem 14) Consider a nested sequence of closed intervals:
$$[a_1, b_1] \supseteq [a_2, b_2] \supseteq [a_3, b_3] \supseteq \cdots$$
Prove that there is a point c which belongs to all of the intervals.
HINT: Let $A = \{a_1, a_2, \ldots\}$ be the set of left endpoints. Show that it has a least upper bound c that belongs to all of the intervals.

(5) Prove Fermat's Theorem.

(6) Suppose $f'(x) = 0$ for all x in an interval. Prove that f is constant on the interval.

For a lot more, click on: MAT-126-old-exams.
I particularly suggest doing the following.
Sample1: 1, 5, 6, 8, 9, 10
F16 MT1: 3, 4, 6, 7
Sample 2: 1 (ignore all log and inverse trig problems for now), 3, 4, 6
F16 MT2: 1, 2, 4, 6
F14 MT2: 1, 2, 3, 5, 6