MATH 126

MIDTERM 1 REVIEW PROBLEMS

Thursday Oct. 5, 2017

NAME (please print legibly): ______________________________

- **No calculators or notes are allowed on this exam.**
- **Please show all of your work. You may use the backs of pages if necessary. You may not receive full credit for a correct answer if there is insufficient work shown.**

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	12	
5	10	
6	12	
7	24	
8	10	
9	6	
TOTAL	104	

1. (10 pts) Prove that $\sqrt{3}$ is irrational.

2. (10 pts) A set S of real numbers is *dense* if, for every pair of real numbers $a < b$, there is an element of S between a and b. For example, $\mathbf{Q}$ is dense because we can find a number between a and b that has a finite decimal expansion, which means it is of the form $\frac{n}{10^m}$ and hence rational. Using the fact that $\mathbf{Q}$ is dense, show that the set of irrational numbers is also dense. (Hint: Given $a < b$, apply the density of $\mathbf{Q}$ to the numbers $a + \sqrt{3} < b + \sqrt{3}$ and then shift back.)

3. (10 pts) Give an example of each of the following, if possible. If not possible, then briefly state why.

(a) Two irrational numbers whose product is rational.

(b) Two irrational numbers whose product is irrational.

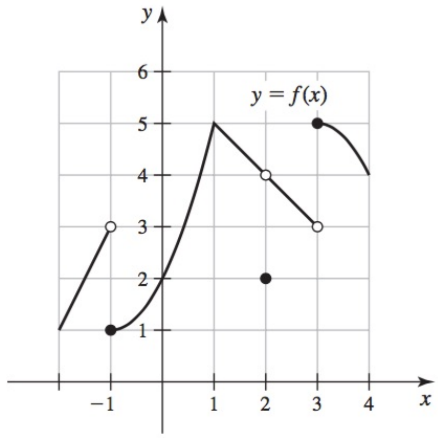
(c) A function defined on all the real numbers but continuous nowhere.

(d) A function $f : \mathbf{N} \longrightarrow \mathbf{N}$ such that $f \circ f = f$, but $f(n) \neq n$ for all n.

4. (12 pts) Prove using mathematical induction that for any $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

5. (10 pts) Let $f(x)$ be the function whose graph is shown below.



For each problem, give the value, or explain why it does not exist.

(a) $\lim_{x \to 1} f(x)$

(b) $\lim_{x \to 2} f(x)$

(c) $f(2)$

(d) $\lim_{x \to 3} f(x)$.

(e) List all values of x at which f fails to be continuous.

6. (12 pts) Evaluate the following limits.

(a) $\lim_{x\to 4} \dfrac{\sqrt{1+2x}-1}{x}$

(b) $\lim_{x\to 0} \dfrac{\sqrt{1+2x}-1}{x}$

(c) $\lim_{h\to 1^-} \dfrac{-1}{h-1}$

7. (24 pts)

(a) Define what it means for $\lim_{x \to a} f(x) = L$.

(b) Show, using the definition of limit, that $\lim_{x \to 4} \frac{x+1}{x+6} = \frac{1}{2}$.

(c) Show, using the definition of limit, that $\lim_{x \to 0} x \cos(\frac{1}{x}) = 0$

(d) Suppose $\lim_{x \to 0} f(x) = 0$, and $g(x)$ is a bounded function (this means that there exists $M > 0$ such that $|g(x)| \leq M$ for all x). Prove using the definition of limit that

$$\lim_{x \to 0} f(x)g(x) = 0.$$

8. (10 pts) The line $y = L$ is a *horizontal asymptote* for a graph $y = f(x)$ if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.
Find all horizontal asymptotes for the graph of the function

$$f(x) = \frac{-3x^2 - 4x^5}{2x + x^5}.$$

9. (6 pts) Suppose $|x - a| < 1$. Prove that $|x + a| < 1 + 2|a|$.