Set 1 → Quantifiers.

a) (Strichartz, §1.1.3, Exercise #2.) For each of the following mathematical statements, rewrite the statement making all the quantifiers explicit. Then form the negation of the statement.

a. Every positive integer has a unique prime factorization.

c. Multiplication of integers is associative.

d. Two points in the plane determine a line.

Set 2 → Cardinality.

a) (Strichartz, §1.2.3, Exercise #5.) Let $A_1, A_2, A_3, ...$ be countable sets, and let their Cartesian product $A_1 \times A_2 \times A_3 \times ...$ be defined to be the set of all sequences $(a_1, a_2, a_3, ...)$, where $a_k$ is an element of $A_k$ for some $k \geq 1$. Prove that the Cartesian product is uncountable. Show that the same conclusion holds if each of the sets $A_1, A_2, A_3, ...$ has at least two elements.

b) Show that the open interval $(0, 1)$ has the same cardinality as $\mathbb{R}$.

Set 3 → Proofs. Prove the following statements.

a) The equality $(1 + 2 + 3 + ... + n)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$ holds for any integer $n > 0$.

b) If $p$ is a prime number, then $\sqrt{p}$ is an irrational number.

c) For every integer $n > 0$, $5^n + (2)3^{n-1} + 1$ is divisible by 8.