Undergraduate Handbook for Mathematics & Statistics

1 Mathematics in Our World

Basic questions we should ask are:

> What is the nature of mathematics?
>
> What role does it play in the modern world?

We hope that a student, interested in mathematics as a major or because of its role in their career path, will ponder these questions guided by the brief reflections given here.
Mathematics has been called the "Queen of the Sciences". History reveals to us the reasons for this and reveals the dual nature of mathematics providing guiding light toward both the above questions. On one hand, mathematics is a content driven discipline constructed via pure mathematical research following the deductive paradigm of axioms, theorems and proofs. This began primarily in the Greek Era over 2000 years ago with the development of Euclidean geometry and overtime has flowered into deep areas of mathematical thought: algebra, analysis, combinatorics, geometry, and number theory to name a few. On the other hand, mathematics is a powerful language to model and understand the physical world. Indeed, mathematics provides a tool in physics providing foundational models in mechanics, thermodynamics, electricity and magnetism, quantum mechanics, and relativity. In the 19th and 20th centuries, the breadth of mathematical applications grew immensely: the industrial age spawned applications of mathematics thoughout the enginneering world; applications in modeling population dynamics, cellular function and dynamics were discovered leading to a diversity of mathematical thought in the biological sciences and medicine; the computer age has

led to new results in discrete mathematics and in applying number theory to encryption algorithms; and most recently mathematics has found significant application in the analysis of financial markets. The meaning of the word "dual" should be clear: often, the results of mathematics provide the tool in the applied world and just as often, the desire to construct a model leads to new mathematical ideas on which to apply the deductive paradigm.
As a creative deductive endeavor, we quote British mathematician Bertrand Russell (circa 1900):

> "Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

As a powerful language for understanding the physical world, we quote French mathematician Henri Poincaré (circa 1890):

> "If one looks at the different problems of the integral calculus which arise naturally when one wishes to go deep into the different parts of physics, it is impossible not to be struck by the analogies existing. Whether it be electrostatics or electrodynamics, the propagation of heat, optics, elasticity, or hydrodynamics, we are led always to the different equations of the same family."

The bottom line of this discussion is that mathematics plays signifcant role in all aspects of modern society. A degree in mathematics or a minor in mathematics opens career paths throughout industry, the physical sciences, the biological sciences and medical profession, business and financial world, the computer science realm, the teaching profession, and the creative endeavor of mathematical discovery.

2 Requirements for the BA Degree Program

Required courses for the BA degree are divided into five categories as follows. *A total of 43 credit hours in mathematics courses is required for a degree. Furthermore, all required courses must be passed with a grade of C or better.*

2.1 The Calculus, 12 credit hours

Anyone who has ever looked at the development of the physical sciences quickly realizes that we understand and measure physical reality by observing change. Indeed, almost all physical laws are expressed in terms of how one quantity changes relative to another. This is the birthplace of The Calculus and the study of Calculus is the first step toward the undergraduate degree in mathematics and many other disciplines. The department offers a three semester sequence in Calculus briefly outlined below.

- MAT 126 *Calculus I*: Here we meet the fundamental concepts of continuity of functions, derivatives, antiderivatives and the definite integral. Various computational skills are developed and a variety of applications of calculus are introduced.

- MAT 127 *Calculus II*: Computational skills for antiderivatives and definite integrals are refined. Further we study power series methods and approximation of functions by polynomials. A brief introduction to differential equations and their application is provided.

- MAT 228 *Multivariate Calculus*: In many ways this is the crowning course of the Calculus sequence. Vectors and vector geometry are developed, functions of several variables along with notions of differentiation and integration are introduced. At the end of the course the fundamental theorems of vector calculus are presented: Green's theorem, the Divergence theorem, and Stokes theorem. It is certainly true that these are fundamental results lying at the foundation of many applications of mathematics in the natural world.

2.2 Core Courses, 16 credit hours

- MAT 261, *Introduction to Abstract Mathematics*. The key purpose of this course is to learn to read and write proofs.

- MAT 262 *Linear Algebra*. A core course offering the theory and applications of matrix algebra.

- MAT 434 *Introduction to Statistics*. A calculus based course introducing the basic ideas in statistics: probability, random variables, distributions, estimation, hypothesis testing, regression and correlation, analysis of variance.

- MAT 425 *Introduction to Real Analysis I*. This treats the underlying theory of The Calculus in detail.
- One of MAT 463 (Intro to Abstract Algebra), MAT 465 (Theory of Numbers), or MAT 481 (Discrete Mathematics)

2.3 Upper Level Mathematics Concentration, 12 credit hours

All mathematics majors must take at least four other approved MAT courses, at least three of which must be at the 400 level or above. These courses should be chosen by the student in consultation with her/his advisor and they should form a coherent area of concentration. Some suggestions are the following. Pure Mathematics, Applied/Interdisciplinary Mathematics, Statistics, Mathematics Education.

2.3.1 Pure Mathematics

Courses in our pure mathematics offerings promote the deductive paradigm of mathematics in particular areas. Put differently, here you will learn to read and write axioms, definitions, lemmas and theorems, and proofs within a particular area of mathematics. Some of our course offerings in this realm are as follows.

- MAT 426 *Real Analysis II* A continuation of MAT 425.
- MAT463/464 *Introduction to Abstract Algebra I/II*. This is a study of abstract axiomatic systems for algebraic structures including the study of homomorphisms, quotient structures and substructures.
- MAT 471 *Differential Geometry* A introduction to the theory of curves and surfaces. A focal point is the notion of curvature and its computational and geometric aspects.
- MAT 475 *Higher Geometry*. Here we learn the axiomatic deductive structure of various geometries.

2.3.2 Applied/Interdisciplinary Mathematics

Mathematics as a language to understand the physical world is an underlying theme in our applied offerings. At the undergraduate level the course offerings are primarily applications of calculus.

- MAT 259 *Differential Equations*. A basic course developing techniques for solving differential equations arising in the physical and biological sciences.
- MAT 453 *Partial Differential Equations*. Geometrical and analytic theory and methods for linear partial equations arising in physics. Particular attention is given equations governing heat flow, wave phenomena, and electrostatic potentials.
- MAT 481 *Discrete Mathematics*. One of our non-calculus based applied courses. Discrete mathematics can be termed the mathematics of computer science. Topics include: graph and network theory, analysis of algorithms, computational complexity.
- MAT 487 *Numerical Analysis* A study of numerical methods in integration, linear algebra, and differential equations.
- MAT 400 *Topics* From time to time the department offers topics courses in applied/applicable mathematics at the 400 level. Some recent titles include: Modeling and Simulation, Vector Analysis, Differential Forms.

2.3.3 Statistics

Statistics is a mathematical science pertaining to the collection, analysis, interpretation or explanation, and presentation of data. It also provides tools for prediction and forecasting based on data. It is applicable to a wide variety of academic disciplines, from the natural and social sciences to the humanities, government and business.

- MAT 435 *Mathematical Statistics* A continuation of MAT 434 with more indepth look at the theory.
- MAT 436 *Nonparametric Statistics* Surveys nonparametics alternatives to standard parametric techniques.

- MAT 437 *Statistical Methods of Research* An introduction to analysis of variance and regression analysis; application and illustrations from many fields.

- MAT 400 *Topics* From time to time the department offers topics courses in statistics at the 400 level. Some recent titles include: Bayesian Statistics, Multivariate Statistics, Actuarial Science.

2.3.4 Mathematics Education

The department offers courses intended to focus the mathematics major for certification as a high school teacher. Examples of these courses are:

- MAT 305 *Mathematics for Secondary School Teachers.* This is a teaching methods course satisfing the state certification requirements.

- MAT 445 *History of Mathematics.* Deals with the lives and times of mathematicians while focusing on mathematical ideas.

- MAT 475 *Higher Geometry*

- MAT 481 *Discrete Mathematics*

2.4 Outside Specialization

Every mathematics major must complete an 18-credit specialization or two 12-credit specializations of advisor approved courses in areas outside of mathematics. The choice of two 12-credit concentrations provides an excellent way to round out a Liberal Arts degree. The combinations are almost endless, e.g., Philosophy and French, Art and Music. Once the decision is made, choices for particular courses in each concentration should be made in consultation with your advisor.
The choice of the 18-credit specialization is a great way to build an outside minor. Below are a few suggested pathways in other areas. Some of these can be made into a minor in the area by taking two additional courses; others are suggested courses which promote a connection with mathematics. Other pathways will be added as time goes on. If you have an interest in another discipline not listed below you should work out a sequence of courses in consultation with your advisor.

2.4.1 Physics Concentration

Physics is the study of matter and its motion through spacetime and all that derives from these, such as energy and force. More broadly, it is the general analysis of nature, conducted in order to understand how the world and universe behave. Physics is both significant and influential, in part because advances in its understanding have often translated into new technologies, but also because new ideas in physics often resonate with the other sciences, mathematics and philosophy.

- PHY 121 and 122: Physics for Engineers and Physical Scientists (8 credits)
- Phy 236: Introduction to Modern Physics (3 credits)
- PHY 238: Mechanics (3 credits)
- PHY 476: Methods of Mathematical Physics (3 credits)
- PHY 454 Electricity and Magnetism I (3 credits) or PHY 462 Physical Thermodynamics (3 credits)

Following up with either PHY 455 Electricity and Magnetism II or PHY 463 Statistical Mechanics would satisfy the minor requirements in Physics.

2.4.2 Biology Concentration

Today's research in the Biological Sciences is strongly oriented towards quantitative and computational methods. Mathematical and computational tools are needed to quantitatively and objectively describe the huge data sets being produced in such realms as bioinformatics in genetics, image processing in cell biology, studies of physiological development.

- BIO 100: Basic Biology (4 credits)
- BIO 200: Biology of Organisms (4 credits)

Choose four courses from the following list:

- BMB 207/208: Fundamentals of Chemistry (4 credits)
- BMB 280: Introduction to Molecular & Cellular Biology

- BMB 300: General Microbiology (3 credits)
- BIO 307: Introduction to Neuroscience (3 credits)
- BIO 310: Plant Biology (3 credits)
- BIO 462: Principles of Genetics (3 credits)

2.4.3 Economics

A solid grasp of the fundamentals of economics is a good start for a career in public life or in the corporate world. As a mathematics major, you are in a great position to relatively quickly acquire a sound knowledge of the more quantitative aspects of the field. Given that the use of quantitative techiniques is increasingly important in economics, a concentration in economics might actually give you an edge over a major in economics when it comes to finding a job.

Option 1

- Required courses:
  - ECO 120 Principles of Microeconomics
  - ECO 121 Principles of Macroeconomics
- One course from the following (grade of C- required):
  - ECO 321 Intermediate Macroeconomic Theory
  - ECO 350 Intermediate Microeconomic Theory
  - ECO 420 Intermediate Microeconomic Theory with Calculus
- Three ECO courses (two of which at the 300 level or above). Any REP courses may count as ECO courses.

Option 2 (Accelerated: 18 credits, at least 9 of which at taken at UMaine)

- Students must obtain a minimum 2.0 grade point average in ECO courses taken pursuant to the requirements of the minor.
- ECO 410 Accelerated Introductory Economics
- One from the following list (with a grade of C- or better):
  - ECO 321 Intermediate Macroeconomics

  - ECO 350 Intermediate Microeconomic Theory
  - ECO 420 Intermediate Microeconomic Theory with Calculus
- Four ECO courses (two of which must be at the 300 level or above). Any REP course may count as ECO courses.

A few notes are in order.

1. For mathematics majors, ECO 420 is strongly suggested.
2. Students who want more quantitative skills and have taken intermediate micro and macro economics should consider taking one or more of the following:
   - ECO 480 Introduction to Mathematical Economics
   - ECO 485 Econometrics
   - ECO 473 Economic and Policy Applications of GIS

2.4.4 Computer Science

Computer science is the study of the theoretical foundations of information and computation. It may be described as the systematic study of algorithmic processes that describe and transform information; the fundamental question underlying computer science is, "What can be efficiently automated?

- COS 120 Introduction to Problem Solving Using Computer Programming
- COS 140 Foundations of Computer Science
- COS 225 Introduction to Object-Oriented Programming and Design
- COS 226 Introduction to Data Structures
- COS 250 Discrete Structures
- One course from the following list:
  - COS 335 Computer Organization and Architecture
  - COS 350 Data Structures and Algorithms
  - COS 415 Computer Simulation and Modeling
  - COS 460 Interactive Computer Graphics
  - COS 470/570 Introduction to Artificial Intelligence

2.4.5 Business

The combination of an undergraduate degree in mathematics and a minor in Business can be a powerful mix for students interested in pursuing a career in the business or corporate world. Two minors are available for UMaine students wishing to complement their mathematics degree with exposure to business as follows.

Business Administration

- BUA 201 Principles of Financial Accounting
- BUA 325 Principles of Managemant and Organization
- BUA 350 Business Finance
- BUA 370 Marketing
- ECO 120 Principles of Microeconomics
- ECO 121 Principles of Macroeconomics
- PSY 100 General Psychology
- BUA elective: One additional business course for which prerequisites have been met.

Accounting

- BUA 201 Principles of Financial Accounting
- BUA 202 Principles of Managerial Accounting
- BUA 301 Intermediate Accounting I
- BUA 302 Intermediate Accounting II
- BUA 305 Cost Accounting
- BUA 310 Auditing
- BUA 312 Federal Taxation of Individuals
- Accounting Elective

2.5 The Capstone Experience

The goal is to pull together various discipline specific threads of your undergraduate program in an experience that typifies professional work in mathematics. This can be achieved in three ways:

- MAT 401 *Capstone Seminar in Mathematics*. This would be taken typically in your senior year. Students write a paper and present an in class talk on a specific topic in mathematics under investigation.

- *Honors Thesis*. For students in the Honors Program, an mathematical honors thesis will replace the requirement of MAT 401.

- *Faculty Research*. Involvement in a significant way with mathematics research being conducted by a faculty member.

3 Suggested Curriculum for the BA in Mathematics

Here is given a suggested outline to pursue the BA degree. The bracketed numbers [x] refer to credit hours. Any courses selected to fall under the heading Electives should bear in mind the University General Education Requirements and the College of Liberal Arts and Sciences distribution requirements (both described below).

- First Year-First Semester
  MAT 126 Calculus I [4]
  ENG 101 College Compositions[3]
  Electives (including General Education Requirements) [7-10]

- First Year-Second Semester
  MAT 127 Calculus II [4]
  Electives [11-14]

- Second Year-First Semester
  MAT 228 Calculus III [4]
  MAT 261 Intro to Abstract Mathematics [3]
  Outside Specialization Course [3]
  Electives [5-8]

- Second Year-Second Semester
  MAT 262 Linear Algebra [3]
  One of MAT 425, 463, 465, 481 [3]
  Outside Specialization course [3]
  Electives [6-9]

- Third Year-First Semester
  MAT 434 Intro to Statistics [4]
  One of MAT 425, 463, 465, 481 [3]
  Outside Specialization course [3]
  Electives [5-8]

- Third Year -Second Semester
  One or two MAT courses for upper-level concentration [3-6]
  Outside specialization course [3]
  Electives [6-9]

- Fourth Year-First Semester
  One or two MAT courses for upper-level concentration [3-6]
  Outside specialization course [3]
  Electives [6-9]

- Fourth Year-Second Semester
  MAT 401 Capstone Seminar [3]
  MAT course for upper-level concentration [3]
  Outside specialization course [3]
  Electives [6-9]

4 The Mathematics Minor Program

The Minor in Mathematics requires completing 24 credits of mathematics courses as follows.
The Calculus Sequence MAT 126, 127, 228 (12 credits)
Any four of the following courses (12 credits):
MAT 258 Intro to Differential Equations with Linear Algebra
MAT 259 Differential Equations
MAT 261 Intro to Abstract Mathematics
MAT 262 Linear Algebra
MAT 300 Introduction to PDE's for Engineers MAT 332 Statistics for Engineers
MAT 425/426 Real Variables I/II
MAT 434 Intro to Statistics
MAT 435 Intro to Mathematical Statistics
MAT 436 Nonparametric Statistics

MAT 437 Statistical Methods in Research
MAT 451 Differential Equations and Dynamical Systems
MAT 453/454 Partial Differential Equations I/II
MAT 456 Network Optimization
MAT 457 Intro to Mathematical Modeling
MAT 463/464 Intro to Abstract Algebra I/II MAT 465 Theory of Numbers
MAT 471 Differential Geometry
MAT 481 Discrete Mathematics
MAT 487 Numerical Analysis
Students who are interested in a Mathematics minor and for whom MAT 258 is required by their major programs are advised to take MAT 259 and MAT 262 (to replace MAT 258). If MAT 258 is selected, nither MAT 259 nor MAT 262 can be used because of overlapping material. *A student must receive a grade of C or higher in all minor requirements.*

5 General Education & College Requirements

Taken from the Undergraduate Catalog, the focus of the General Education Requirements is stated thus:

> The University's goal is to ensure that all of its graduates, regardless of the academic major they pursued, are broadly educated persons who can appreciate the achievements of civilization, understand the tensions within it, and contribute to resolving them.

Overall, general education takes up approximately one-third of any program and is meant to be flexible in the manner that its goals are met. The six categories are as follows.

- *Science.* Two courses in the physical or biological sciences. This can be accomplished in two ways: (a) By completing two courses with laboratory component in the basic or applied sciences. (b) By completing one laboratory course as above and a second approved course that incorporates laboratory experience and applications of scientific knowledge.

- *Human Values and Social Context.* This amounts to 18 credits selected from lists of approved courses. Sub-categories, from which one course from each is required are: (a) Western cultural tradition. (b) Social context and institutions. (c) Cultural diversity and international

perspectives. (d) Population and the environment. (e) Artistic and creative expression.

- *Mathematics.* Six credit hours in mathematics or statistics or certain courses in computer science. (this is not a issue for mathematics majors)

- *Writing Competency.* Nine credit hours are required. This includes: (a) ENG 101, College Composition (grade of C or better) or its equivalent on the basis of a placement exam or certain Honors courses (111 or 112 with a grade of C or better); (b) Two courses designated as writing intensive, at least one of which must be within the academic major. In mathematics, the latter is MAT 261, Introduction to Abstract Mathematics.

- *Ethics.* At least one approved course placing substantial emphasis on the discussion of ethical issues.

- *Capstone Experience.* This has be described previously.

On the outset, the General Education requirement amount to approximately 47-50 credit hours. However, overlap is possible within the requirements among the categories, e.g., the ethics requirement or writing competency may fall under Human Values and Social Context. Typically, the general education requirements can be achieved with 40 credit hours.
In addition to the General Education requirements, for the BA degree the College requires nine credits in courses numbered 200 or above with at least one course prerequisite in an area outside their academic major. These cannot overlap with the Gen Ed courses taken. Two points to mention are:

1. The college requirements can be built upon Gen Ed courses.

2. The mathematics degree program requires outside specialization described previously. This can overlap with the College requirements.

6 Create a Four Year Plan

Toward the end of your first semester or in the beginning of your second semester at UMaine the following is a worthwhile educational experience/life exercise:

Keeping in mind your mathematical interests, your interests outside of mathematics, the General Education requirements, and College requirements, design your four year program including what courses will be taken each semester. You may use the template given in the third section as a guide. Do not worry about the fact that you do not know what courses will be offered and when, substitutions and rearrangements can be made as necessary. Focus on you and your educational goals, such a plan will help you keep on track. After completing your plan to a degree, show and discuss it with your advisor. Make sure she/he keeps a copy.