

Problems for First Year Students

1. Prof. Bray lives S miles from university and it takes him T minutes to drive from home to work. Prove that at some instant the magnitude of the acceleration of his car is at least $\frac{4S}{T^2}$.
2. Explain how to construct a pair of concave up functions, whose product is concave down.
3. How does the number of real roots of the equation $e^x = ax^2$ depend on the parameter a ?
4. Find a one-to-one function from $[0, 1]$ onto $(0, 1)$.
5. Suppose polynomials $p(x)$ and $q(x)$ have integer values for exactly the same values of x . Prove that either $p(x) - q(x)$ or $p(x) + q(x)$ is a constant.
6. Prove that a hexagon $ABC A' B' C'$ with parallel opposite sides can be inscribed in a circle if and only if its diagonals AA' , BB' and CC' are equal.
7. Prove that $x^9 - 6x^7 + 9x^5 - 4x^3$ is divisible by 8640 for all integer values of x .
8. Let $n = 5^{(2^{98})}$. Prove that the last 100 decimal digits of n and n^2 are the same.
9. The consecutive odd integers are grouped as follows: $\{1\}$, $\{3,5\}$, $\{7,9,11\}$, $\{13,15,17,19\}$,...Find, with proof, the sum of the numbers in the n th group
10. Let $p(x)$ be a polynomial with rational coefficients. Prove that $p(\sqrt[3]{2})$ is a root of a polynomial with rational coefficients of degree at most 3.