Mathematics Undergraduate Handbook

Herein we present an overview of mathematics and what it takes to major or minor in mathematics at the University of Maine.

1 Mathematics in Our World

Basic questions we should ask are:

Whats is the nature of mathematics as a discipline?

What role does it play in the modern world?

We hope that a student interested in mathematics as a major or because of its role in their career path will ponder these questions guided by the brief reflections given here.
Mathematics has been called the "Queen of the Sciences". History reveals to us the reasons for this and reveals the dual nature of mathematics providing guiding light toward both the above questions. On one hand, mathematics is a content driven discipline constructed via pure mathematical research following the deductive paradigm of axioms, theorems and proofs. This began primarily in the Greek Era over 2000 years ago with the development of Euclidean geometry and overtime has flowered into deep areas of mathematical thought: algebra, analysis, combinatorics, geometry, and number theory to name a few. On the other hand, mathematics is a powerful language to model and understand the physical world. Indeed, mathematics provides a tool in physics providing foundational models in mechanics, thermodynamics, electricity and magnetism, quantum mechanics, and relativity. In the 19th and 20th centuries, the breadth of mathematical applications grew immensely: the industrial age spawned applications of mathematics thoughout

the enginneering world; applications in modeling population dynamics, cellular function and dynamics were discovered leading to a diversity of mathematical thought in the biological sciences and medicine; the computer age has led to new results in discrete mathematics and in applying number theory to encryption algorithms; and most recently mathematics has found significant application in the analysis of financial markets. The meaning of the word "dual" should be clear: often, the results of mathematics provide the tool in the applied world and just as often, the desire to construct a model leads to new mathematical ideas on which to apply the deductive paradigm.
As a creative deductive endeavor, we quote British mathematician Bertrand Russell (circa 1900):

> "Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."

As a powerful language for understanding the physical world, we quote French mathematician Henri Poincaré (circa 1890):

> "If one looks at the different problems of the integral calculus which arise naturally when one wishes to go deep into the different parts of physics, it is impossible not to be struck by the analogies existing. Whether it be electrostatics or electrodynamics, the propagation of heat, optics, elasticity, or hydrodynamics, we are led always to the different equations of the same family."

The bottom line of this discussion is that mathematics as a discipline plays signifcant role in all aspects of modern society. A degree in mathematics or a minor in mathematics opens career paths throughout industry, the physical sciences, the biological sciences and medical profession, business and financial world, the computer science realm, the teaching profession, and the creative endeavor of mathematical discovery.

2 Mathematical Stepping Stones

We present a brief overview of the path to a mathematics degree. Mathematics courses listed below are for illustrative purposes of the stepping stones.

Other mathematics courses are available and more complete descriptions of courses listed below can be found in the Undergraduate Catalog.

2.1 The Calculus

Anyone who has ever looked at the development of the physical sciences quickly realizes that we understand and measure physical reality by observing change. Indeed, almost all physical laws are expressed in terms of how one quantity changes relative to another. This is the birthplace of The Calculus and the study of Calculus is the first step toward the undergraduate degree in mathematics and many other disciplines. The department offers a three semester sequence in Calculus briefly outlined below.

- MAT 126 *Calculus I*: Here we meet the fundamental concepts of continuity of functions, derivatives, antiderivatives and the definite integral. Various computational skills are developed and you will be introduced to a variety of applications of calculus.

- MAT 127 *Calculus II*: Computational skills for antiderivatives and definite integrals are refined. Further we study power series methods and approximation of functions by polynomials. A brief introduction to differential equations and their application is provided.

- MAT 228 *Multivariate Calculus*: In many ways this is the crowning course of the Calculus sequence. Vectors and vector geometry are developed, functions of several variables along with notions of differentiation and integration. At the end of the course the fundamental theorems of vector calculus are presented: Green's theorems, the Divergence theorem, and Stokes theorem. It is certainly true that these are fundamental results lying at the foundation of many applications of mathematics in the natural world.

2.2 Abstract Mathematics

Courses in our abstract offerings promote the deductive paradigm of mathematics in particular content areas. Put differently, here you will learn to read and write axioms, definitions, lemmas and theorems, and proofs within a particular area of mathematics. Some of our course offerings in the abstract realm are as follows.

- MAT 261, *Introduction to Abstract Mathematics*. The key purpose of this course is to learn to read and write proofs.

- MAT 425/426 *Introduction to Real Analysis I/II*. This treats the underlying theory of The Calculus in detail.

- MAT463/464 *Introduction to Abstract Algebra I/II*. This is a study of abstract axiomatic systems for algebraic structures and binary relations.

- MAT 475 *Higher Geometry*. Here we learn the axiomatic deductive structure of various geometries.

2.3 Applied Mathematics & Statistics

Mathematics as a language to understand the physical world is an underlying theme in our applied and statistics offerings. At the undergraduate level the course offerings are primarily applications of calculus.

- MAT 259 *Differential Equations*. A basic course developing techniques for solving differential equations arising in the physical and biological sciences.

- MAT 262 *Linear Algebra*. A core course offering the theory and applications of matrix algebra.

- MAT 434 *Introduction to Statistics*. A calculus based course introducing the basic ideas in statistics: probability, random variables, distributions, estimation, hypothesis testing, regression and correlation, analysis of variance.

- MAT 453 *Partial Differential Equations*. Geometrical and analytic theory and methods for linear partial equations arising in physics. Particular attention is given equations governing heat flow, wave phenomena, and electrostatic potentials.

- MAT 481 *Discrete Mathematics*. One of our non-calculus based applied courses. Discrete mathematics can be termed the mathematics of computer science. Topics include: graph and network theory, analysis of algorithms, computational complexity.

2.4 Mathematics Education

The department offers courses intended to focus the mathematics major for certification as a high school teacher. Examples of these courses are:

- MAT 305 *Mathematics for Secondary School Teachers*. This is a teaching methods course satisfing the state certification requirements.
- MAT 445 *History of Mathematics*. Deals with the lives and times of mathematicians while focusing on mathematical ideas.
- MAT 475 and 481 (listed previously)

2.5 The Capstone Experience

The goal is to pull together various discipline specific threads of your undergraduate program in an experience that typifies professional work in mathematics. This can be achieved in three ways:

- MAT 401 *Capstone Seminar in Mathematics*. This would be taken typically in your senior year. Students write a paper and present an in class talk on a specific topic in mathematics under investigation.
- *Honors Thesis*. For students in the Honors Program, an mathematical honors thesis will replace the requirement of MAT 401.
- *Faculty Research*. Involvement in a significant way with mathematics research being conducted by a faculty member.

3 The BA Degree Program

3.1 Requirements

Required courses for the BA degree are divided into three areas as follows. Numbers in brackets [x] represent credit hours. *A total of 43 credit hours in mathematics courses is required for a degree. Furthermore, all required courses must be passed with a grade of C or better.*

Core Courses, 31 credit hours

- The Calculus Sequence: MAT 126, 127, 228 [12]

- MAT 261 Introduction to Abstract Mathematics [3]
- MAT 262 Linear Algebra [3]
- MAT 434 Introduction to Statistics [4]
- MAT 425 Introduction to Real Variables I [3]
- MAT 401 Capstone Seminar in Mathematics [3]
- One of MAT 463 (Intro to Abstract Algebra), MAT 465 (Theory of Numbers), or MAT 481 (Discrete Mathematics) [3]

Upper Level Mathematics Concentration, 12 credit hours At least four other approved MAT courses, at least three of which must be at the 400 level or above. These courses should be chosen by the student in consultation with her/his advisor and they should form a coherent area of concentration, e.g., Pure Mathematics, Applied Mathematics, Statistics, Mathematics Education.

Outside Specialization Every mathematics major must complete an 18-credit specialization or two 12-credit specializations of advisor approved courses in areas outside of mathematics. *The choice of the 18-credit specialization is a great way to build an outside minor.*

3.2 Suggested Curriculum for the BA in Mathematics

Here is given a suggested outline to pursue the BA degree. The bracketed numbers [x] refer to credit hours. Any courses selected to fall under the heading Electives should bear in mind the University General Education Requirements and the College of Liberal Arts and Sciences distribution requirements (both described below).

- First Year-First Semester
  LAS 100 Majoring in the Liberal Arts and Sciences [1]
  MAT 126 Calculus I [4]
  ENG 101 College Compositions[3]
  Electives (including General Education Requirements) [7-10]

- First Year-Second Semester
  MAT 127 Calculus II [4]
  Electives [11-14]

- Second Year-First Semester
  MAT 228 Calculus III [4]
  MAT 261 Intro to Abstract Mathematics [3]
  Outside Specialization Course [3]
  Electives [5-8]

- Second Year-Second Semester
  MAT 262 Linear Algebra [3]
  One of MAT 425, 463, 465, 481 [3]
  Outside Specialization course [3]
  Electives [6-9]

- Third Year-First Semester
  MAT 434 Intro to Statistics [4]
  One of MAT 425, 463, 465, 481 [3]
  Outside Specialization course [3]
  Electives [5-8]

- Third Year -Second Semester
  One or two MAT courses for upper-level concentration [3-6]
  Outside specialization course [3]
  Electives [6-9]

- Fourth Year-First Semester
  One or two MAT courses for upper-level concentration [3-6]
  Outside specialization course [3]
  Electives [6-9]

- Fourth Year-Second Semester
  MAT 401 Capstone Seminar [3]
  MAT course for upper-level concentration [3]
  Outside specialization course [3]
  Electives [6-9]

4 The Mathematics Minor Program

The Minor in Mathematics requires completing 24 credits of mathematics courses as follows.
The Calculus Sequence MAT 126, 127, 228 (12 credits)

Any four of the following courses (12 credits):
MAT 258 Intro to Differential Equations with Linear Algebra
MAT 259 Differential Equations
MAT 261 Intro to Abstract Mathematics
MAT 262 Linear Algebra
MAT 300 Introduction to PDE's for Engineers MAT 332 Statistics for Engineers
MAT 425/426 Real Variables I/II
MAT 434 Intro to Statistics
MAT 435 Intro to Mathematical Statistics
MAT 436 Nonparametric Statistics
MAT 437 Statistical Methods in Research
MAT 451 Differential Equations and Dynamical Systems
MAT 453/454 Partial Differential Equations I/II
MAT 456 Network Optimization
MAT 457 Intro to Mathematical Modeling
MAT 463/464 Intro to Abstract Algebra I/II MAT 465 Theory of Numbers
MAT 471 Differential Geometry
MAT 481 Discrete Mathematics
MAT 487 Numerical Analysis
Students who are interested in a Mathematics minor and for whom MAT 258 is required by their major programs are advised to take MAT 259 and MAT 262 (to replace MAT 258). If MAT 258 is selected, nither MAT 259 nor MAT 262 can be used because of overlapping material. *A student must receive a grade of C or higher in all minor requirements.*

5 General Education & College Requirements

Taken from the Undergraduate Catalog, the focus of the General Education Requirements is stated thus:

> The University's goal is to ensure that all of its graduates, regardless of the academic major they pursued, are broadly educated persons who can appreciate the achievements of civilization, understand the tensions within it, and contribute to resolving them.

Overall, general education takes up approximately one-third of any program and is meant to be flexible in the manner that its goals are met. The six

categories are as follows.

- *Science.* Two courses in the physical or biological sciences. This can be accomplished in two ways: (a) By completing two courses with laboratory component in the basic or applied sciences. (b) By completing one laboratory course as above and a second approved course that incorporates laboratory experience and applications of scientific knowledge.

- *Human Values and Social Context.* This amounts to 18 credits selected from lists of approved courses. Sub-categories, from which one course from each is required are: (a) Western cultural tradition. (b) Social context and institutions. (c) Cultural diversity and international perspectives. (d) Population and the environment. (e) Artistic and creative expression.

- *Mathematics.* Six credit hours in mathematics or statistics or certain courses in computer science. (this is not a issue for mathematics majors)

- *Writing Competency.* Nine credit hours are required. This includes: (a) ENG 101, College Composition (grade of C or better) or its equivalent on the basis of a placement exam or certain Honors courses (111 or 112 with a grade of C or better); (b) Two courses designated as writing intensive, at least one of which must be within the academic major. In mathematics, the latter is MAT 261, Introduction to Abstract Mathematics.

- *Ethics.* At least one approved course placing substantial emphasis on the discussion of ethical issues.

- *Capstone Experience.* This has be described previously.

On the outset, the General Education requirement amount to approximately 47-50 credit hours. However, overlap is possible within the requirements among the categories, e.g., the ethics requirement or writing competency may fall under Human Values and Social Context. Typically, the general education requirements can be achieved with 40 credit hours.
In addition to the General Education requirements, for the BA degree the College requires nine credits in courses numbered 200 or above with at least one course prerequisite in an area outside their academic major. These cannot overlap with the Gen Ed courses taken. Two points to mention are:

1. The college requirements can be built upon Gen Ed courses.

2. The mathematics degree program requires outside specialization described previously. This can overlap with the College requirements.

6 Create a Four Year Plan

Toward the end of your first semester or in the beginning of your second semester at UMaine the following is a worthwhile educational experience/life exercise:

> Keeping in mind your mathematical interests, your interests outside of mathematics, the General Education requirements, and College requirements, design your four year program including what courses will be taken each semester. You may use the template given in the third section as a guide. Do not worry about the fact that you do not know what courses will be offered and when, substitutions and rearrangements can be made as necessary. Focus on you and your educational goals, such a plan will help you keep on track. After completing your plan to a degree, show and discuss it with your advisor. Make sure she/he keeps a copy.